ON REGULARIZABLE BIRATIONAL MAPS

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ABSTRACT. Bedford asked if there exists a birational self map f of the complex projective plane such that for any automorphism A of the complex projective plane $A\circ f$ is not conjugate to an automorphism. In this article we give such an f of degree 5.

1. INTRODUCTION

Let us consider $\mathrm{Bir}(\mathbb{P}^k_{\mathbb{C}})$ the group of all birational self maps of $\mathbb{P}^k_{\mathbb{C}}$, also called the *$k$-dimensional Cremona group*. A birational map $f\in\mathrm{Bir}(\mathbb{P}^k_{\mathbb{C}})$ is *regularizable* if there there exist a smooth projective variety V and a birational map $g\colon V \dashrightarrow \mathbb{P}^k_{\mathbb{C}}$ such that $g^{-1}\circ f\circ g$ is an automorphism of V. For instance any $f\in\mathrm{Aut}(\mathbb{P}^k_{\mathbb{C}})=\mathrm{PGL}(k+1,\mathbb{C})$ is regularizable but any $f\in\mathrm{Bir}(\mathbb{P}^2_{\mathbb{C}})$ such that $(\deg f^n)_n$ grows linearly is not regularizable ([6]). To any element f of $\mathrm{Bir}(\mathbb{P}^k_{\mathbb{C}})$ we associate the set $\mathrm{Reg}(f)$ defined by

$$\mathrm{Reg}(f) := \big\{A\in\mathrm{Aut}(\mathbb{P}^k_{\mathbb{C}}) \mid A\circ f \text{ is regularizable}\big\}.$$

On the one hand Dolgachev asked[1] whether there exists a birational self map of $\mathbb{P}^k_{\mathbb{C}}$ of degree >1 such that $\mathrm{Reg}(f)=\mathrm{Aut}(\mathbb{P}^k_{\mathbb{C}})$. In [4] we give a negative answer to this question; more precisely we prove:

Theorem 1.1 ([4]). *Let $\Bbbk$ be an uncountable, algebraically closed field. Let f be a birational self map of $\mathbb{P}^m_{\Bbbk}$ of degree $d\geq 2$. The set of automorphisms A of $\mathbb{P}^m_{\Bbbk}$ such that* $\deg\big((A\circ f)^n\big)\neq\big(\deg(A\circ f)\big)^n$ *for some $n>0$ is a countable union of proper Zariski closed subsets of* $\mathrm{PGL}(m+1,\Bbbk)$.

Let $\Bbbk$ be a field of characteristic zero. Let f be a birational transformation of $\mathbb{P}^m$ which is defined over the field $\Bbbk$, i.e. the formulas defining f have coefficients in $\Bbbk$. Then, there exists an element A of $\mathrm{PGL}(m+1,\Bbbk)$ *such that* $\deg((A\circ f)^n)=\deg(f)^n$ *for all $n\geq 1$.*

On the other hand Bedford asked: does there exist a birational map f of $\mathbb{P}^k_{\mathbb{C}}$ such that $\mathrm{Reg}(f)=\emptyset$? We will focus on the case $k=2$. According to [1, 5] if $f\in\mathrm{Bir}(\mathbb{P}^2_{\mathbb{C}})$ and

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$\deg f = 2$, then $\mathrm{Reg}(f) \neq \emptyset$. Furthermore Blanc proves that the set

$$\big\{f \in \mathrm{Bir}(\mathbb{P}^2_{\mathbb{C}}) \,|\, \deg f = 3,\, \mathrm{Reg}(f) \neq \emptyset,\, \lim_{n\to+\infty} (\deg(f^n))^{1/n} > 1\big\}$$

is dense in $\big\{f \in \mathrm{Bir}(\mathbb{P}^2_{\mathbb{C}}) \,|\, \deg f = 3\big\}$ and that its complement has codimension 1 (*see* [2]). Blanc also gives a positive answer to Bedford question in dimension 2: if $\chi \in \mathrm{Bir}(\mathbb{P}^2_{\mathbb{C}})$ is given by

$$\chi\colon (x:y:z) \dashrightarrow \big(xz^5 + (yz^2+x^3)^2 : yz^5 + x^3z^3 : z^6\big)$$

then $\mathrm{Reg}(\chi) = \emptyset$. Note that $\chi = (x+y^2,y)\circ(x,y+x^3)$ in the affine chart $z=1$. Indeed Blanc example can be generalized as follows: the birational map of degree np given in the affine chart $z=1$ by

$$\chi_{n,p} = (x+y^n,y)\circ(x,y+x^p) = \big(x+(y+x^p)^n, y+x^p\big)$$

satisfies $\mathrm{Reg}(\chi_{n,p}) = \emptyset$. Finally Blanc asked ([2, Question 1.6]): does there exists $f \in \mathrm{Bir}(\mathbb{P}^2_{\mathbb{C}})$ such that $\deg f < 6$ and $\mathrm{Reg}(f) = \emptyset$? In this article we give a positive answer:

Theorem A. *If ψ is the birational self map of $\mathbb{P}^2_{\mathbb{C}}$ given by*

$$\psi\colon (x:y:z) \dashrightarrow \big(x^2yz^2 - z^5 + x^5 : x^2(x^2y - z^3) : xz(x^2y-z^3)\big),$$

then $\mathrm{Reg}(\psi) = \emptyset$.

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2. PROOF OF THEOREM A

Let S be a smooth projective surface. Let $\phi\colon S \dashrightarrow S$ be a birational map. This map admits a resolution

$$\begin{tikzcd} & Z \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_2"] & \\ S \arrow[rr, dashed, "\phi"'] & & S, \end{tikzcd}$$

where $\pi_1 : Z \to S$ and $\pi_2 : Z \to S$ are finite sequences of blow-ups. The resolution is *minimal* if and only if no (-1)-curve of Z is contracted by both π_1 and π_2. Assume ϕ is minimal; the *base-points* $\mathrm{Base}(\phi)$ of ϕ are the points blown-up by π_1, which can be points of S or infinitely near points. Finally we denote by $\mathrm{Exc}(\phi)$ the set of curves contracted by ϕ.

Denote by $\mathfrak{b}(\phi)$ the number of base-points of ϕ; note that $\mathfrak{b}(\phi)$ is equal to the difference of the ranks of $\mathrm{Pic}(Z)$ and $\mathrm{Pic}(S)$ and thus equal to $\mathfrak{b}(\phi^{-1})$. Let us introduce the *dynamical* *number of the base-points of* ϕ

$$\mu(\phi)=\lim_{k\to+\infty}\frac{\mathfrak{b}(\phi^k)}{k}.$$

Since $\mathfrak{b}(\phi\circ\varphi)\leq\mathfrak{b}(\phi)+\mathfrak{b}(\varphi)$ for any birational self map φ of S, $\mu(\phi)$ is a non-negative real number. As $\mathfrak{b}(\phi)=\mathfrak{b}(\phi^{-1})$ one gets $\mu(\phi^k)=|k\mu(\phi)|$ for any $k\in\mathbb{Z}$. Furthermore if Z is a smooth projective surface and $\varphi\colon S\dashrightarrow Z$ a birational map, then for all $n\in\mathbb{Z}$

$$-2\mathfrak{b}(\varphi)+\mathfrak{b}(\phi^n)\leq\mathfrak{b}(\varphi\circ\phi^n\circ\varphi^{-1})\leq 2\mathfrak{b}(\varphi)+\mathfrak{b}(\phi^n);$$

hence $\mu(\phi)=\mu(\varphi\circ\phi\circ\varphi^{-1})$. One can thus state the following result:

Lemma 2.1 ([3])**.** *The dynamical number of base-points is an invariant of conjugation. In particular if* ϕ *is a regularizable birational self map of a smooth projective surface, then* $\mu(\phi)=0$.

A base-point p of ϕ is a *persistent base-point* if there exists an integer N such that for any $k\geq N$

- ◇ $p\in\mathrm{Base}(\phi^k)$
- ◇ and $p\not\in\mathrm{Base}(\phi^{-k})$.

Let p be a point of S or a point infinitely near S such that $p\not\in\mathrm{Base}(\phi)$. Consider a minimal resolution of ϕ

$$\begin{tikzcd} & Z \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_2"] & \\ S \arrow[rr, dashed, "\phi"'] & & S. \end{tikzcd}$$

Because p is not a base-point of ϕ it corresponds via π_1 to a point of Z or infinitely near; using π_2 we view this point on S again maybe infinitely near and denote it $\phi^\bullet(p)$. For instance if $S=\mathbb{P}^2_\mathbb{C}$, $p=(1:0:0)$ and f is the birational self map of $\mathbb{P}^2_\mathbb{C}$ given by

$$(z_0:z_1:z_2)\dashrightarrow(z_1z_2+z_0^2:z_0z_2:z_2^2)$$

the point $f^\bullet(p)$ is not equal to $p=f(p)$ but is infinitely near to it. Note that if φ is a birational self map of S and p is a point of S such that $p\not\in\mathrm{Base}(\phi)$, $\phi(p)\not\in\mathrm{Base}(\varphi)$, then $(\varphi\circ\phi)^\bullet(p)=\varphi^\bullet(\phi^\bullet(p))$. One can put an equivalence relation on the set of points of S or infinitely near S: the point p is *equivalent* to the point q if there exists an integer k such that $(\phi^k)^\bullet(p)=q$; in particular $p\not\in\mathrm{Base}(\phi^k)$ and $q\not\in\mathrm{Base}(\phi^{-k})$. Remark that the equivalence class is the generalization of set of orbits for birational maps.

Let us give the relationship between the dynamical number of base-points and the equivalence classes of persistent base-points:

Proposition 2.2 ([3])**.** *Let S be a smooth projective surface. Let ϕ be a birational self map of S.*

Then $\mu(\phi)$ coincides with the number of equivalence classes of persistent base-points of ϕ. In particular $\mu(\phi)$ is an integer.

This interpretation of the dynamical number of base-points allows to prove the following result that gives a characterization of regularizable birational maps:

Theorem 2.3 ([3])**.** *Let ϕ be a birational self map of a smooth projective surface. Then ϕ is regularizable if and only if $\mu(\phi) = 0$.*

2.1. **Base-points of ψ.** The birational map

$$\psi\colon (x:y:z) \dashrightarrow \big(x^2yz^2 - z^5 + x^5 : x^2(x^2y - z^3) : xz(x^2y - z^3)\big)$$

has only one base-point in $\mathbb{P}^2_{\mathbb{C}}$, namely $p_1 = (0:1:0)$, and all its base-points are in tower that is: the nine base-points of ψ that we denote p_1, p_2, …, p_9 are such that p_i is infinitely near to p_{i-1} for $2 \leq i \leq 9$. We denote by $\pi\colon S \to \mathbb{P}^2_{\mathbb{C}}$ the blow up of the 9 base-points, and still write L_x (resp. $\mathcal{C}$) the strict transform of the line $L_x \subset \mathbb{P}^2_{\mathbb{C}}$ of equation $x = 0$ (resp. the curve of equation $x^2y - z^3 = 0$) which is contracted by ψ. We denote by $E_i \subset S$ the strict transform of the curve obtained by blowing up p_i. The configuration of the curves E_1, E_2, …, E_9, L_x and $\mathcal{C}$ is

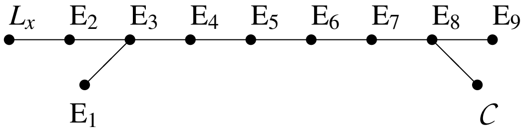

FIGURE 1

Two curves are connected by an edge if their intersection is positive. Let us write $\psi_A = A \circ \psi$ where A is an automorphism of $\mathbb{P}^2_{\mathbb{C}}$. Because π is the blow-up of the base-points of ψ, which are also the base-points of ψ_A, the map $\eta = \psi_A \circ \pi$ is a birational morphism $S \to \mathbb{P}^2_{\mathbb{C}}$ which is the blow-up of the base-points of ψ_A^{-1}. In fact this diagram

$$\begin{tikzcd} & S \arrow[dl, "\pi"'] \arrow[dr, "\eta"] & \\ \mathbb{P}^2_{\mathbb{C}} \arrow[rr, dashed, "\psi_A"'] & & \mathbb{P}^2_{\mathbb{C}} \end{tikzcd}$$

is the minimal resolution of ψ_A.

The morphism η contracts L_x and $\mathcal{C}$ as well as the union of seven other irreducible curves which are among the curves E_1, E_2, …, E_9. The configuration of Figure 1 shows that η contracts the curves L_x, E_2, E_3, E_4, E_5, E_6, E_7, E_8, $\mathcal{C}$ following this order.

We can see $\eta\colon S \to \mathbb{P}^2_{\mathbb{C}}$ as a sequence of nine blow-ups in the same way as we did for π. We denote by q_1, q_2, …, q_9 the base-points of ψ_A^{-1} (or equivalently the points blown up by η) so that $q_1 \in \mathbb{P}^2_{\mathbb{C}}$ and q_i is infinitely near to q_{i-1} for $2 \leq i \leq 9$. We denote by $D \subset \mathbb{P}^2_{\mathbb{C}}$ (resp. $\mathcal{C}' \subset \mathbb{P}^2_{\mathbb{C}}$) the line contracted by ψ_A^{-1} which is the image by A of the line $y = 0$ (resp. of the conic $z^2 - xy = 0$). We denote by $F_i \subset S$ the strict transform of the curve obtained by blowing up q_i. Because of the order of the curves contracted by η we get equalities between L_x, $\mathcal{C}$, E_1, E_2, …, E_9 and D, $\mathcal{C}'$, F_1, F_2, …, F_9 as follows

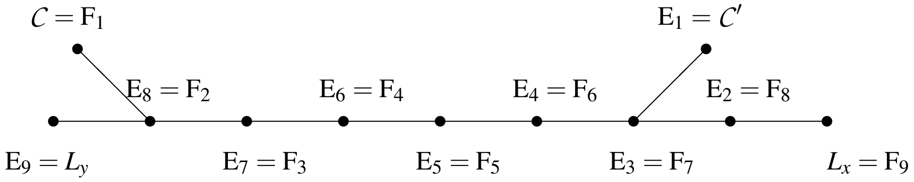

FIGURE 2

In particular we see that the configuration of the points q_1, q_2, …, q_9 is not the same as that of the points p_1, p_2, …, p_9. Saying that a point m is proximate to a point m' if m is infinitely near to m' and that it belongs to the strict transform of the curve obtained by blowing up m' the configurations of the points p_i and q_i are

$$\begin{tikzcd}
p_1 & p_2 \arrow[l] & p_3 \arrow[l] \arrow[ll, bend left] & p_4 \arrow[l] & p_5 \arrow[l] & p_6 \arrow[l] & p_7 \arrow[l] & p_8 \arrow[l] & p_9 \arrow[l]
\end{tikzcd}$$

$$\begin{tikzcd}
q_1 & q_2 \arrow[l] & q_3 \arrow[l] & q_4 \arrow[l] & q_5 \arrow[l] & q_6 \arrow[l] & q_7 \arrow[l] & q_8 \arrow[l] & q_9 \arrow[l]
\end{tikzcd}$$

FIGURE 3.

An arrow corresponds to the relation "is proximate to"

We will prove that for any integer $i > 0$ the point p_3 belongs to $\mathrm{Base}(\psi_A^i)$ and does not belong to $\mathrm{Base}(\psi_A^{-i})$. It implies that $\mu(\psi_A) > 0$ and that ψ_A is not regularizable.

Denote by k the lowest positive integer such that p_1 belongs to $\mathrm{Base}(\psi_A^{-k})$. If no such integer exists we write $k=\infty$. For any $1\leq i<k$ the point p_1 does not belong to $\mathrm{Base}(\psi_A^{-i})$ so ψ_A and ψ_A^{-1} have no common base-point. As a consequence the set of base-points of the map $\psi_A^{i+1}=\psi_A\circ\psi_A^i$ is the union of the base-points of ψ_A^i and of the points $(\psi_A^{-i})^\bullet(p_j)$ for $1\leq j\leq 9$. Since the map ψ_A^{-i} is defined at p_1 the point $(\psi_A^{-i})^\bullet(p_j)$ is proximate to the point $(\psi_A^{-i})^\bullet(p_k)$ if and only if p_j is proximate to p_k. Proceeding by induction on i we get the following assertions:

- ◇ $\mathrm{Base}(\psi_A^i)=\{(\psi_A^{-m})^\bullet(p_j)\,|\,1\leq j\leq 9, 0\leq m\leq i-1\}$ for any $1\leq i\leq k$;
- ◇ for any $0\leq -\ell\leq k$ the configuration of the points $\{(\psi_A^\ell)^\bullet(p_j)\,|\,1\leq j\leq 9\}$ is given by

$$\begin{tikzcd}
(\psi_A^\ell)^\bullet(p_1) & (\psi_A^\ell)^\bullet(p_2) \arrow[l] & (\psi_A^\ell)^\bullet(p_3) \arrow[l] \arrow[ll, bend left] & (\psi_A^\ell)^\bullet(p_4) \arrow[l] & (\psi_A^\ell)^\bullet(p_5) \arrow[l] & (\psi_A^\ell)^\bullet(p_6) \arrow[l] & (\psi_A^\ell)^\bullet(p_7) \arrow[l] & (\psi_A^\ell)^\bullet(p_8) \arrow[l] & (\psi_A^\ell)^\bullet(p_9) \arrow[l]
\end{tikzcd}$$

Hence the point p_3 belongs to $\mathrm{Base}(\psi_A^i)$ for any $1\leq i\leq k$.

If $k=\infty$, then p_3 belongs to $\mathrm{Base}(\psi_A^i)$ for any $i>0$ and by definition of k the point p_1 does not belong to $\mathrm{Base}(\psi_A^{-i})$ for any $i>0$, and so neither p_3. We can thus assume that k is a positive integer.

Assume that q_1 belongs to $\mathrm{Base}(\psi_A^i)$ for some $1\leq i\leq k-1$. Then q_1 is equal to $(\psi_A^{-m})^\bullet(p_j)$ for some $0\leq m\leq k-2$ and $1\leq j\leq 9$. This implies that p_j belongs to $\mathrm{Base}(\psi_A^{m+1})$ which is impossible because $m+1\leq k-1$. Hence q_1 does not belong to $\mathrm{Base}(\psi_A^i)$ for any $1\leq i\leq k-1$.

We thus see that ψ_A^{-1} has no common base-point with ψ_A^i for $1\leq i\leq k-1$. In particular if B denotes $\mathrm{Base}(\psi_A^{-1})\cap\mathrm{Base}(\psi_A^k)$, then

$$B=\{(\psi_A^{-(k-1)})^\bullet(p_j)\,|\,1\leq j\leq 9\}\cap\{q_j\,|\,1\leq j\leq 9\}.$$

Let us remark that p_1 belongs to $\mathrm{Base}(\psi_A^{-k})$ and p_1 does not belong to $\mathrm{Base}(\psi_A^{-(k-1)})$; as a result $(\psi_A^{-(k-1)})^\bullet(p_1)$, which is a base-point of ψ_A^k, is also a base-point of ψ_A^{-1}. The set B is thus not empty.

The configurations of the two sets of points $\{(\psi_A^{-(k-1)})^\bullet(p_j)\,|\,1\leq j\leq 9\}$ and $\{q_j\,|\,1\leq j\leq 9\}$ imply that $q_1=(\psi_A^{-(k-1)})^\bullet(p_1)$.

Moreover either $B=\{q_1\}$, or $B=\{q_1,q_2\}$. Indeed $(\psi_A^{-(k-1)})^\bullet(p_3)$ is proximate to $(\psi_A^{-(k-1)})^\bullet(p_2)$ and $(\psi_A^{-(k-1)})^\bullet(p_1)$ whereas q_3 is proximate to q_2 but not to q_1.

The point $(\psi_A^{-(k-1)})^\bullet(p_3)$ is thus a point infinitely near to q_1 in the second neighborhood which is maybe infinitely near to q_2 but not equal to q_3. Recalling that η is the blow up of $q_1, q_2, \ldots, q_9$ the point $(\eta^{-1}\circ\psi_A^{-(k-1)})^\bullet(p_3)$ corresponds to a point that belongs, as a proper or infinitely near point, to one of the curves $F_1, F_2\subset S$. So $(\pi\circ\eta^{-1}\circ\psi_A^{-(k-1)})^\bullet(p_3)$ is a point infinitely near to p_3. For any $1 \leq i \leq k$ the point p_3 does not belong to $\mathrm{Base}(\psi_A^{-i})$; therefore there is no base-point of ψ_A^{-i} which is infinitely near to p_3. As a result $(\psi_A^{-k})^\bullet(p_3)$ does not belong to $\mathrm{Base}(\psi_A^{-i})$ and p_3 does not belong to $\mathrm{Base}(\psi_1^{-(k+i)})$. Moreover $(\psi_A^{-(k+i)})^\bullet(p_3)$ is infinitely near to $(\psi_A^{-i})^\bullet(p_3)$. Choosing $i = k$ we see that $(\psi_A^{-2k})^\bullet(p_3)$ is infinitely near to $(\psi_A^{-k})^\bullet(p_3)$ which is infinitely near to p_3. Continuing like this we get

$$\forall i \geq 1 \qquad p_3 \not\in \mathrm{Base}(\psi_A^{-i}).$$

To get the result it remains to show that p_3 belongs to $\mathrm{Base}(\psi_A^i)$ for any $i \geq 1$. Reversing the order of ψ_A and ψ_A^{-1} we prove as previously that

$$\forall i \geq 1 \qquad q_3 \not\in \mathrm{Base}(\psi_A^i).$$

Let us now see that

$$\big(\forall i \geq 1 \quad q_3 \not\in \mathrm{Base}(\psi_A^i)\big) \Rightarrow \big(\forall i \geq 1 \quad p_3 \in \mathrm{Base}(\psi_A^i)\big).$$

For $i = 1$ it is obvious. Assume $i > 1$; let us decompose

- ◇ ψ_A^i into $\psi_A^{i-1} \circ \psi_A$,
- ◇ $\pi\colon S \to \mathbb{P}^2_\mathbb{C}$ into $\pi_{12} \circ \pi_{39}$ where $\pi_{12}\colon Y \to \mathbb{P}^2_\mathbb{C}$ is the blow up of p_1, p_2 and $\pi_{39}\colon S \to Y$ is the blow up of p_3, p_4, $\ldots$, p_9,
- ◇ $\eta\colon S \to \mathbb{P}^2_\mathbb{C}$ into $\eta_{12} \circ \eta_{39}$ where $\eta_{12}\colon Z \to \mathbb{P}^2_\mathbb{C}$ is the blow up of q_1, q_2 and $\eta_{39}\colon S \to Z$ is the blow up of q_3, q_4, $\ldots$, q_9.

Note that η_{39} contracts F_9, F_8, $\ldots$, F_3 onto the point $Z \ni q_3 \not\in \mathrm{Base}(\psi_A^{i-1} \circ \eta_{12})$. Consider the system of conics of $\mathbb{P}^2_\mathbb{C}$ passing through p_1, p_2 and p_3. Denote by Λ its lift on Y; it is a system of smooth curves passing through q_3 with movable tangents and $\dim \Lambda = 2$. The strict transform of Λ on S is a system of curves intersecting E_3 at a general movable point. The map η_{39} contracts the curves L_x, E_2, E_3, E_4, E_5, E_6, E_7. As the curve E_3 is contracted and is not the last one, the image of the system by η_{39} passes through q_3 with a fixed tangent corresponding to the point q_4. Since $q_3 \not\in \mathrm{Base}(\psi_A^{i-1} \circ \eta_{12})$ the image of $\Lambda \subset Y$ by $\psi_A^{i-1} \circ \eta \circ (\pi_{39})^{-1}$ has a fixed tangent at the point $(\psi_A^{i-1} \circ \eta_{12})(q_3)$. As a consequence p_3 belongs to $\mathrm{Base}(\psi_A^{i-1} \circ \eta \circ (\pi_{39})^{-1})$ and thus to $\mathrm{Base}(\underbrace{\psi_A^{i-1} \circ \eta \circ (\pi_{39})^{-1} \circ (\pi_{12})^{-1}}_{\psi_A^i})$.

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